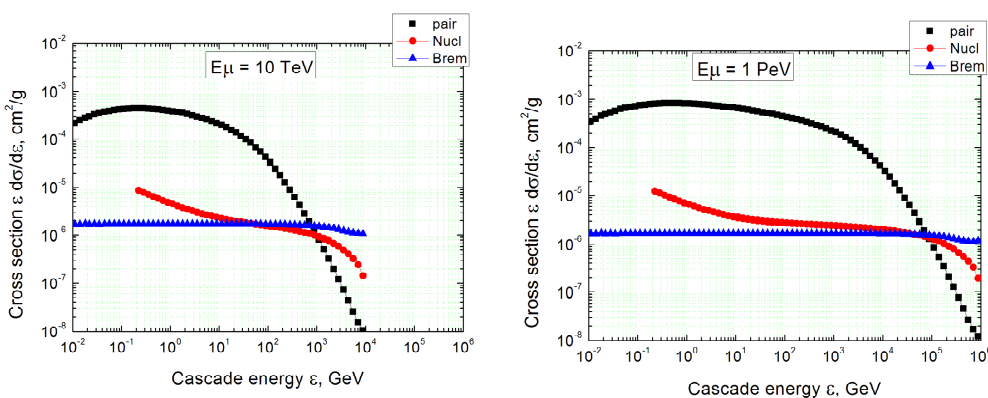


## Introduction

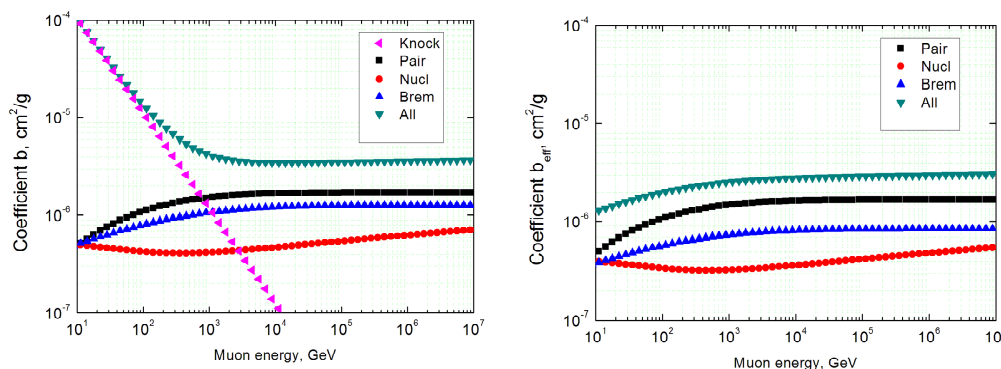
Modern neutrino observatories are deployed in the lakes, seas and glaciers of Antarctica (BNT-200+ (GVD), ANTARES (KM3Net), IceCube) and reach an effective volume of 1 km<sup>3</sup>. Their work is based on the detection of Cherenkov radiation that occurs when relativistic charged particles pass through the water of a detector.

Although the detectors are built to register neutrinos, they have special triggers for the registration of the muon component, and the total exposure time is several years. The combination of these factors provides a unique opportunity to measure the spectrum of cascade showers and to obtain an estimate of the parameters of the spectrum of muons at very high energies.

## Cross sections of processes

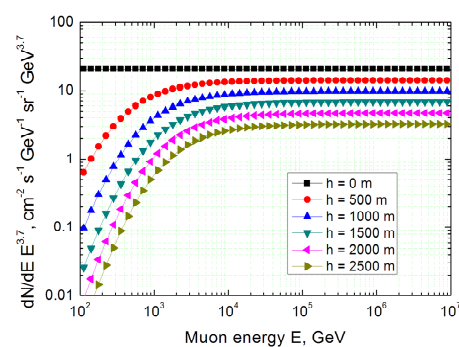


## Coefficients b and b\_eff ( $\gamma = 2.7$ )

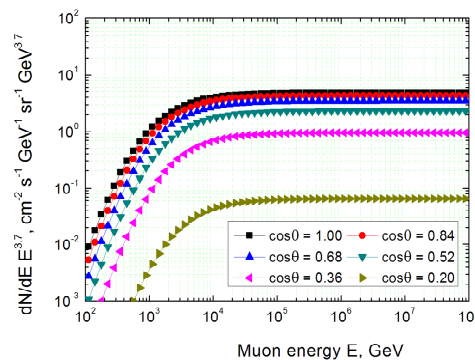


## Calculated muon spectra

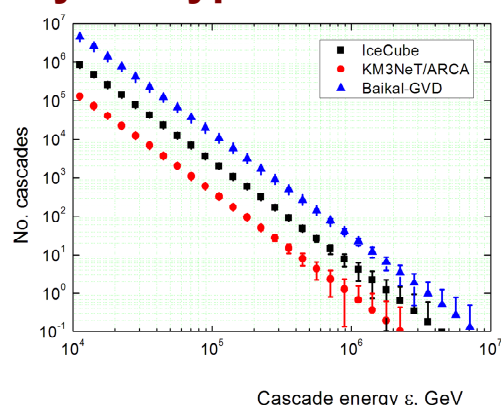
Muon spectra for different depths for the vertical direction



Muon spectra for different zenith angles at the depth of 2000 m



## Estimated integral cascade spectra for 1 year hypothetical runtime



## Approach to muon spectra calculation

Average specific muon energy losses:

$$-(dE/dx) = a + bE$$

Connection of muon energy on the surface and muon energy at depth x:

$$E_0 = \exp(bx) \left( E + \frac{a}{b} \right) - \frac{a}{b}$$

Assuming a simple power law muon spectrum at the surface:

$$N(E, 0) = AE^{-(\gamma+1)}$$

we can calculate muon spectrum at the depth x:

$$N(E, x) = A \exp(-\gamma bx) \left\{ E + \frac{a}{b} [1 - \exp(-bx)] \right\}^{-(\gamma+1)}$$

But it is necessary to take into account energy loss fluctuations. It can be done by means of two additional coefficients  $b_{\text{eff}_1}$  and  $b_{\text{eff}_2}$ :

$$N(E, x) = A \exp(-\gamma b_{\text{eff}_1} x) \left\{ E + \frac{a}{b_{\text{eff}_2}} [1 - \exp(-b_{\text{eff}_2} x)] \right\}^{-(\gamma+1)}$$

## Calculation of spectra of muon-induced cascades

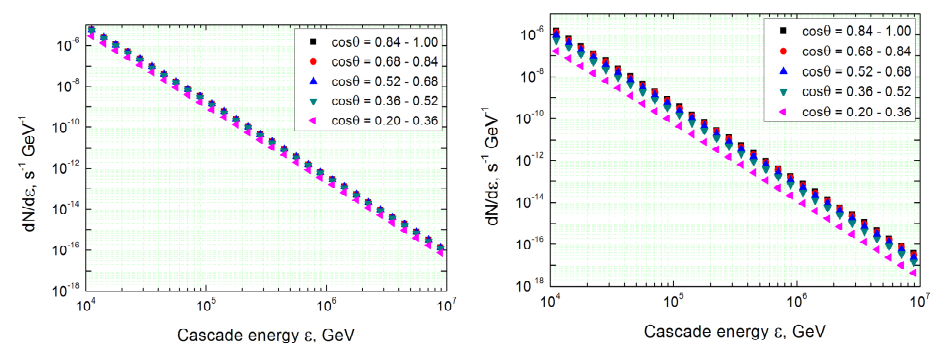
Differential spectrum of cascades at the depth x ( $x = \rho h / \cos\theta$ ) for zenith angles  $\theta$ :

$$N(\epsilon, x, \cos\theta) = M \cdot \sum_I \int_{E_{\text{min}}}^{\infty} N_I(E, x, \cos\theta) \cdot \sigma_I(E, \epsilon) dE$$

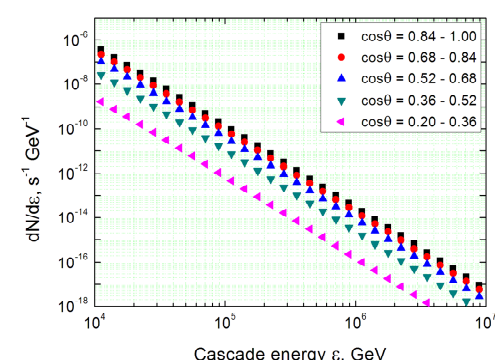
where  $M$  is the mass of the target;  $N(E, x, \cos\theta)$  is the differential spectrum of muons;  $\sigma_I(E, \epsilon)$  are the differential cross sections of generation of cascades with energy  $\epsilon$  by muon with energy  $E$  in three processes (pair production, muon bremsstrahlung and inelastic muon interaction with nuclei).

## Calculated differential cascade spectra for different ranges of zenith angles

Baikal-GVD (h = 650 - 1250 m) IceCube (h = 1500 - 2400 m)



KM3Net/ARCA (h = 3250 - 3750 m)



## Conclusion

Neutrino telescopes such as IceCube, GVD and KM3Net allow to measure spectra of stochastic energy losses of muons (cascades) inside cubic kilometer water volumes. These spectra are closely related with muon energy spectrum. The ratio of cascade spectra at different zenith angles is sensitive to the shape of the muon spectrum at the surface.

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