ISCRA 2019

Simple geometrical model of nucleus-nucleus interactions at VHE

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Introduction

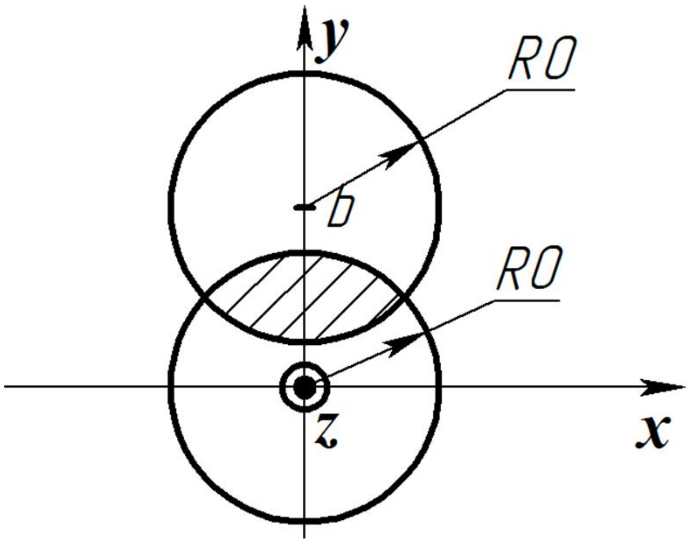
- We consider nucleus-nucleus collisions in Cosmic Rays in a pure geometrical approach
- We set up a few questions:

1. How many nucleons participate in the collisions?
2. What is the energy of the QGP-blob produced in the collisions?
3. How can we estimate the angular momentum of the interacting part of the nuclei?
4. What part of the nucleons determine the energy we measure on the Earth's surface?
5. How can we estimate the speed of the QGP-blob, produced in the collision?

Model

- Hard sphere
- Number of participants in the collision is proportional to the volume of intersection, also it depends on the impact parameter b (the distance between the centers of the nuclei)
- $\Delta A = \Delta A_1 + \Delta A_2$
- $\Delta A_i / A_i = \Delta V_i / V_i$
- $R \sim A^{1/3}$
- We are considering two types of nuclei collisions:
    1. Equal nuclei
    2. Different nuclei

Equal nuclei
(A_1=A_2=A_0)



Equation of the sphere

$$x^2 + y^2 + z^2 = R_0^2; \qquad z(x,y) = \sqrt{R_0^2 - x^2 - y^2}.$$

Equation of the projections

$$x^2 + y^2 = R_0^2\,; \qquad y_2 = \sqrt{R_0^2 - x^2};$$

$$x^2 + (y-b)^2 = R_0^2; \qquad y_1 = b - \sqrt{R_0^2 - x^2}.$$

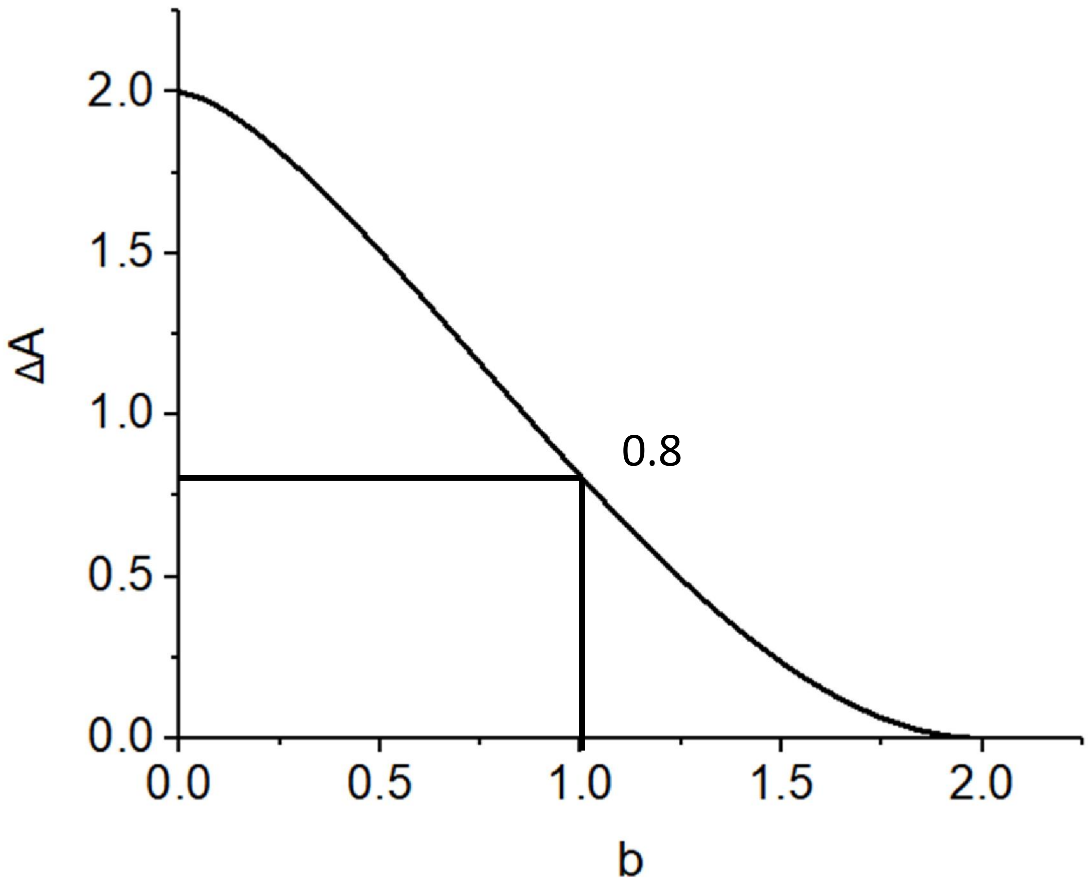
Cross-section volume

$$\Delta V = \int_{-x_0}^{x_0} dx \int_{y_1(x)}^{y_2(x)} dy \int_{-z\,(x,y)}^{z(x,y)} dz\,;$$

$$\Delta V(b) = 2\int_{-x_0}^{x_0} dx \int_{b-\sqrt{R_0^2 - x^2}}^{\sqrt{R_0^2 - x^2}} \sqrt{R_0^2 - x^2 - y^2}\, dy.$$

Fraction of nucleons in the region of interaction of two equal nuclei with $R_1 = R_2 = 1$ and $A_1 = A_2 = 1$

$0 < b < 2R$ $0 < \Delta A < 2A_0$



Influence of probability of collisions with various b

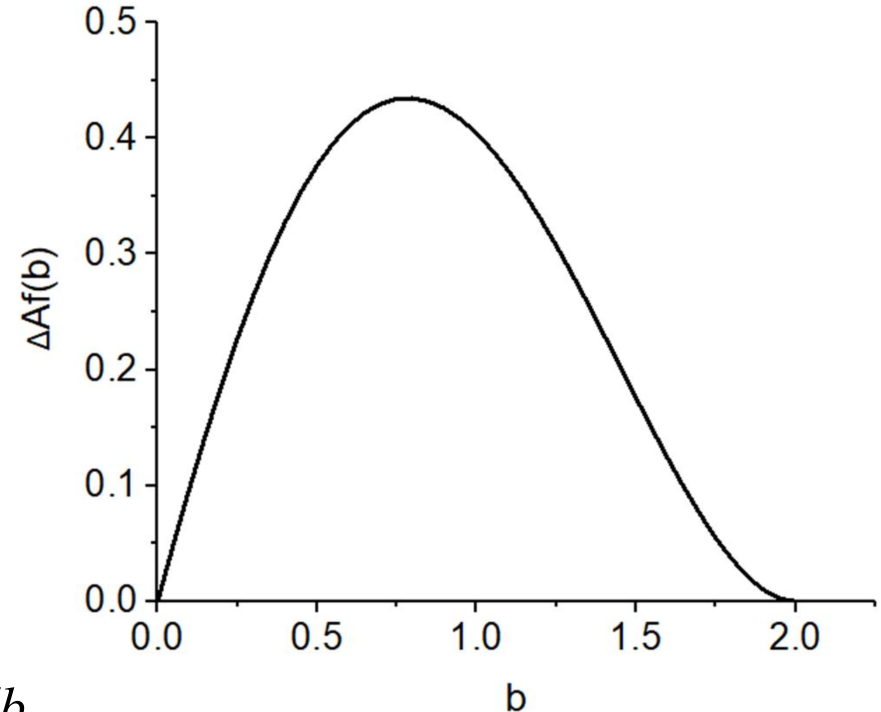
$$d\sigma = 2\pi b db$$

$$\sigma_{full} = \pi (2R_0)^2 = 4\pi R_0^2$$

$$dP = \frac{d\sigma}{\sigma_{full}} = \frac{b}{2R_0^2} db$$

$$f(b) = \frac{b}{2R_0^2}$$

$$\langle \Delta A \rangle = \int \Delta A dP = \int_0^{2R_0} \Delta A f(b) db$$



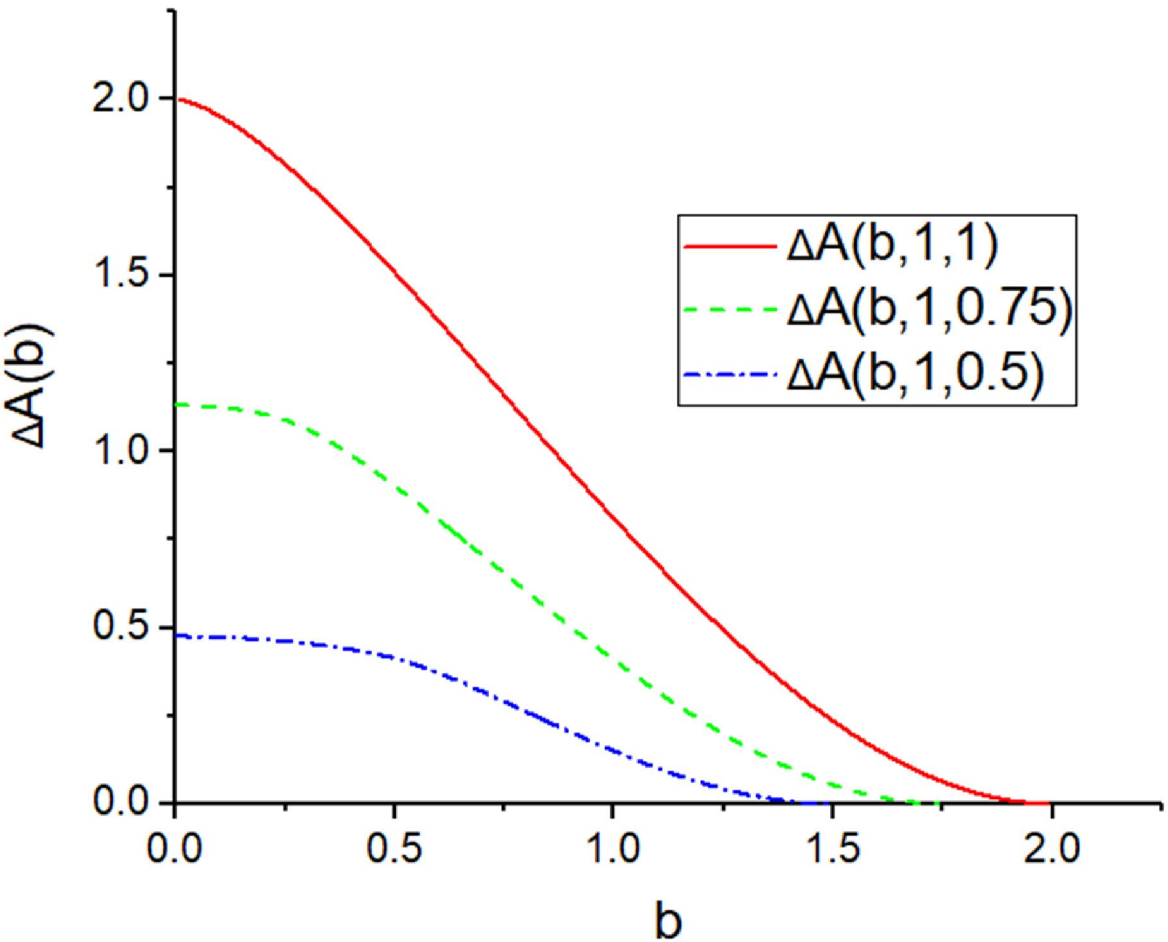
$$\langle \Delta A \rangle = \frac{A_0}{2} \quad (!)$$

Only 25 % nucleons of each nucleus participate in the collision!

Collisions of two different nuclei

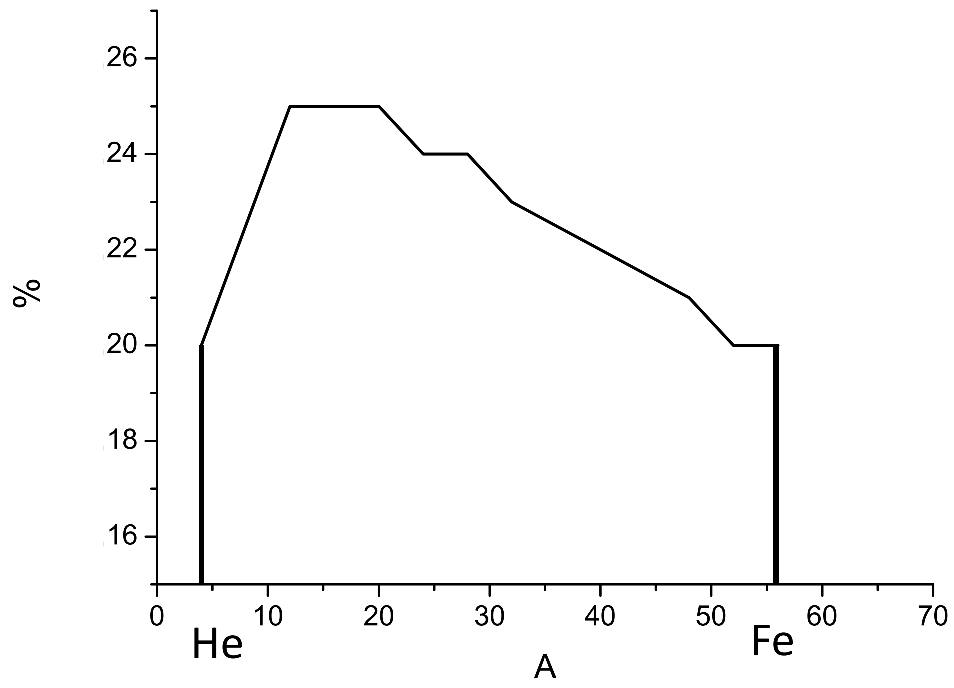
- There are three regions for two unequal nuclei:
$0 < b < R_1 - R_2 < b < \sqrt{R_1^2 - R_2^2} < b < R_1 + R_2$
- The formulas may look a bit complicated(but the approach is the same), so we only present the results of the calculation

Fraction of the nucleons $\Delta A(b, R_1, R_2)$



Interaction of cosmic ray nuclei with the atmosphere of the Earth

CR nucleus	Number of nucleons (A)	The average number of interacting nucleons: In CR nucleus	In nucleus of the atmosphere	Simultaneously in the CR nucleus and nucleus atmosphere CR	frac
He	4	1.47	2.25	3.72	**0.20**
C	12	3.19	3.40	6.59	**0.25**
O	16	3.87	3.74	7.61	**0.25**
Ne	20	4.48	4.02	8.50	**0.25**
Mg	24	5.04	4.26	9.29	**0.24**
Si	28	5.56	4.46	10.02	**0.24**
S	32	6.04	4.64	10.68	**0.23**
Ar	40	6.93	4.94	11.87	**0.22**
Ca	40	6.93	4.94	11.87	**0.22**
Ti	48	7.74	5.19	12.93	**0.21**
Cr	52	8.12	5.30	13.42	**0.20**
Fe	**56**	**8.49**	**5.41**	**13.90**	**0.20**



$\sqrt{S}$ distribution

- Let A_2 be the number of the nucleons in the atmosphere nucleus, A_2=14.5
- Let A_1 be the number of the nucleons in the incident nucleus
- Then we will get:

$$\sqrt{s} = \sqrt{2\Delta m_2 \Delta E_1}$$

$$\Delta m_2 = \Delta A_2 m_N$$

$$\Delta E_1 = E_1 \frac{\Delta A_1}{A_1}$$

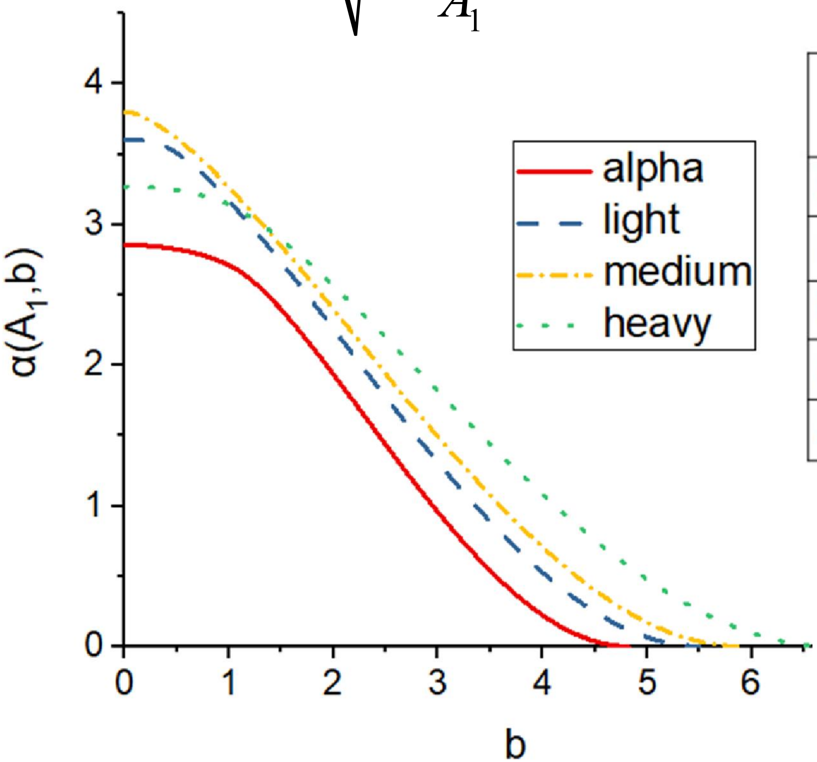
$$\sqrt{s_0} = \sqrt{2 m_N E_1}$$

$$\sqrt{s} = \sqrt{s_0}\sqrt{\frac{\Delta A_1 \Delta A_2}{A_1}}$$

$\sqrt{S}$ distribution

- Let us define a new function:

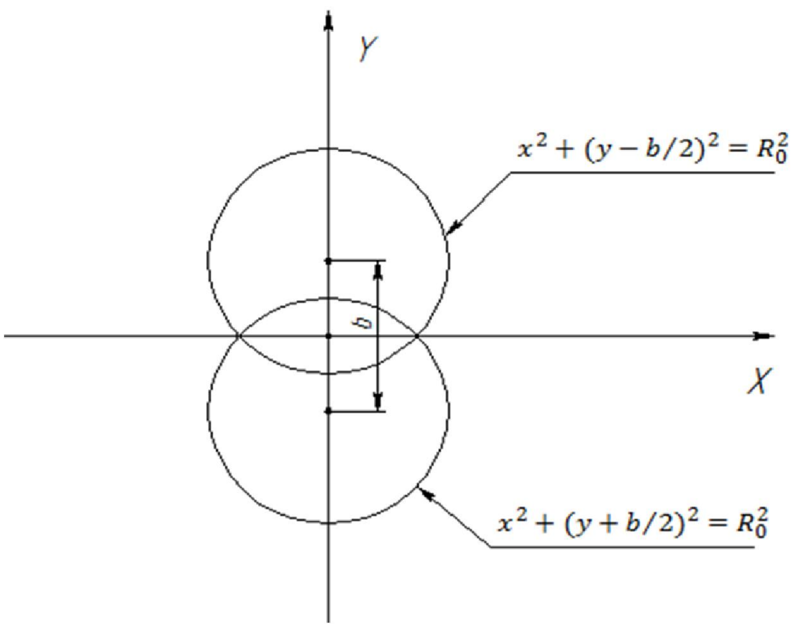
$$\alpha(A_1, b) = \sqrt{\frac{\Delta A_1 \Delta A_2}{A_1}}$$

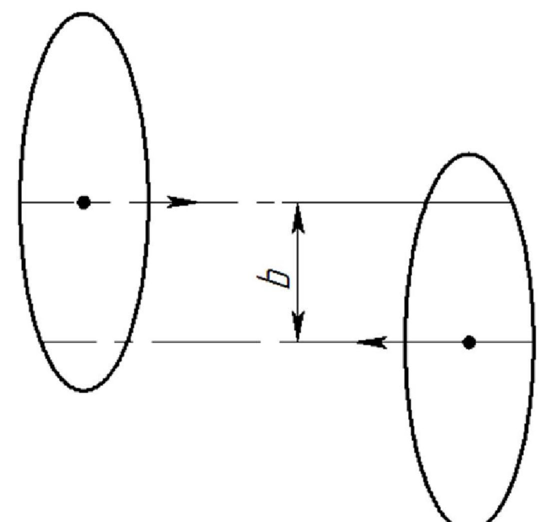


Particles	Z	$\langle A \rangle$	Energy per nucleon	Energy per nucleus
Protons	*1*	*1*	*92 %*	*~40 %*
α – particles	*2*	*4*	*7 %*	*~20 %*
Light nuclei	*3 – 5*	*10*	*0.15 %*	*~1 %*
Medium nuclei	*6 – 10*	*15*	*0.5 %*	*~18 %*
Heavy nuclei	*≥11*	*32*	*0.15 %*	*~18 %*

Angular momentum for the interacting part of nuclei (two equal nuclei)

- Angular momentum $\overrightarrow{L}$ is defined as $\vec{L} = [\vec{r} \times \vec{p}]$
- Here let us consider that we know the value of the momentum of one particle $\overrightarrow{p}_{nucl}$



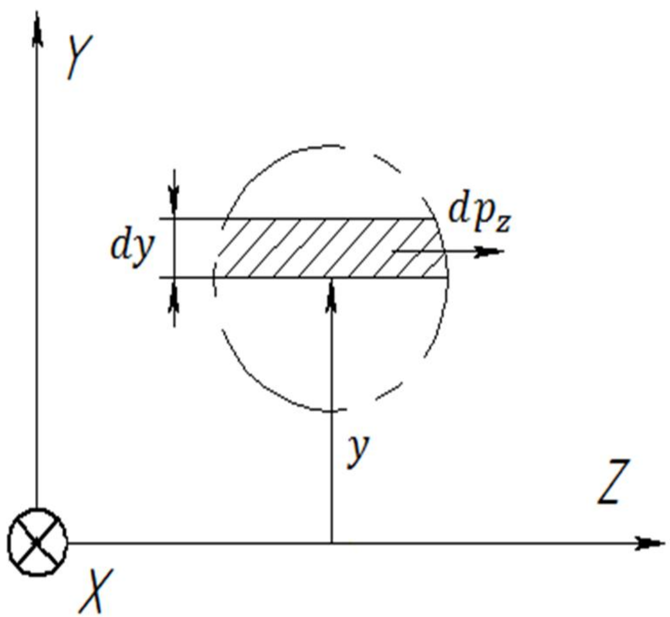


Angular momentum for the interacting part of nuclei (two equal nuclei)

- We choose the projection of the angular momentum on the x-axis, which is the most important:

$$dL_x = y dp_z(y, b)$$

$$\frac{dp_z(y,b)}{dy} = p_{nucl}\left\{\left(\frac{d\mathrm{A}}{dy}\right)^1 - \left(\frac{d\mathrm{A}}{dy}\right)^2\right\}$$



- $\left(\frac{d\mathrm{A}}{dy}\right)^1$ −describes the incident nucleus
- $\left(\frac{d\mathrm{A}}{dy}\right)^2$ −describes the target

Angular momentum for the interacting part of nuclei (two equal nuclei)

- We see that equations of the projections on XY-plane for the incident and target nuclei respectively are

$$x^2 + (y - b/2)^2 = R_0^2 \qquad\qquad x^2 + (y + b/2)^2 = R_0^2$$

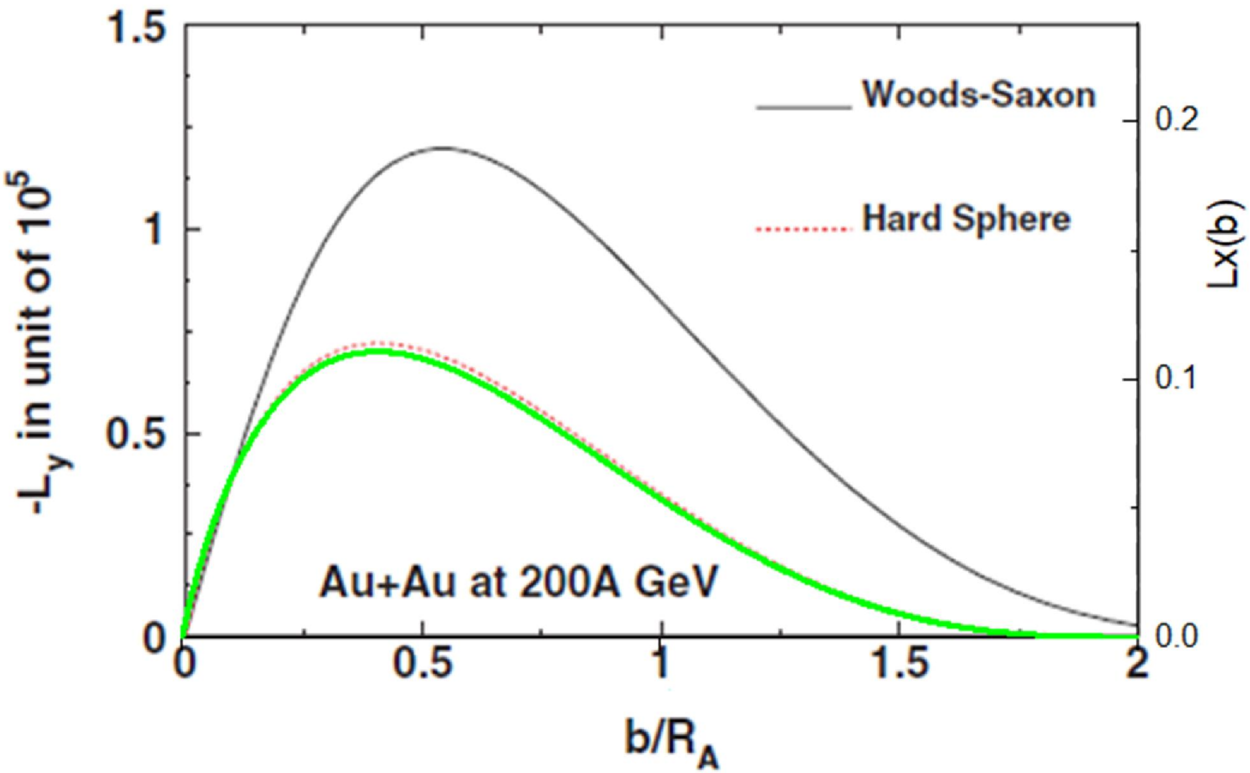
- So: $\left(\frac{d\mathrm{A}}{dy}\right)^1 = \int_{-x_{max}}^{x_{max}} dx \int_{-z(x,y)}^{z(x,y)} dz$, where $z(x,y) = \sqrt{R_0^2 - x^2 - y^2}$

- We get a result:

$$L_x = \int_{-y_{max}}^{y_{max}} y\, p_{nucl} \left\{ \left(\frac{d\mathrm{A}}{dy}\right)^1 - \left(\frac{d\mathrm{A}}{dy}\right)^2 \right\} dy$$

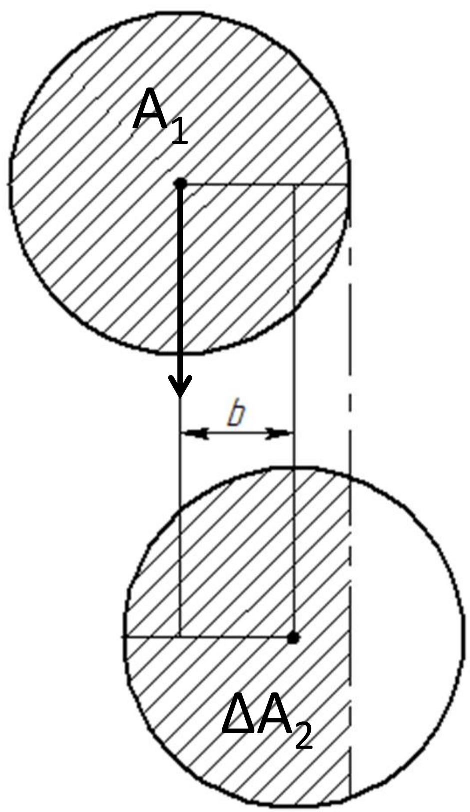
- Let us set p_{nucl} equal to 1, then we get a dependence of the angular momentum on the impact parameter

Angular momentum for the interacting part of nuclei (two equal nuclei)



Jian-Hua Gao et al., Phys. Rev. C 77 (2008) 044902

Number of nucleons which fly towards the Earth's surface after the collision

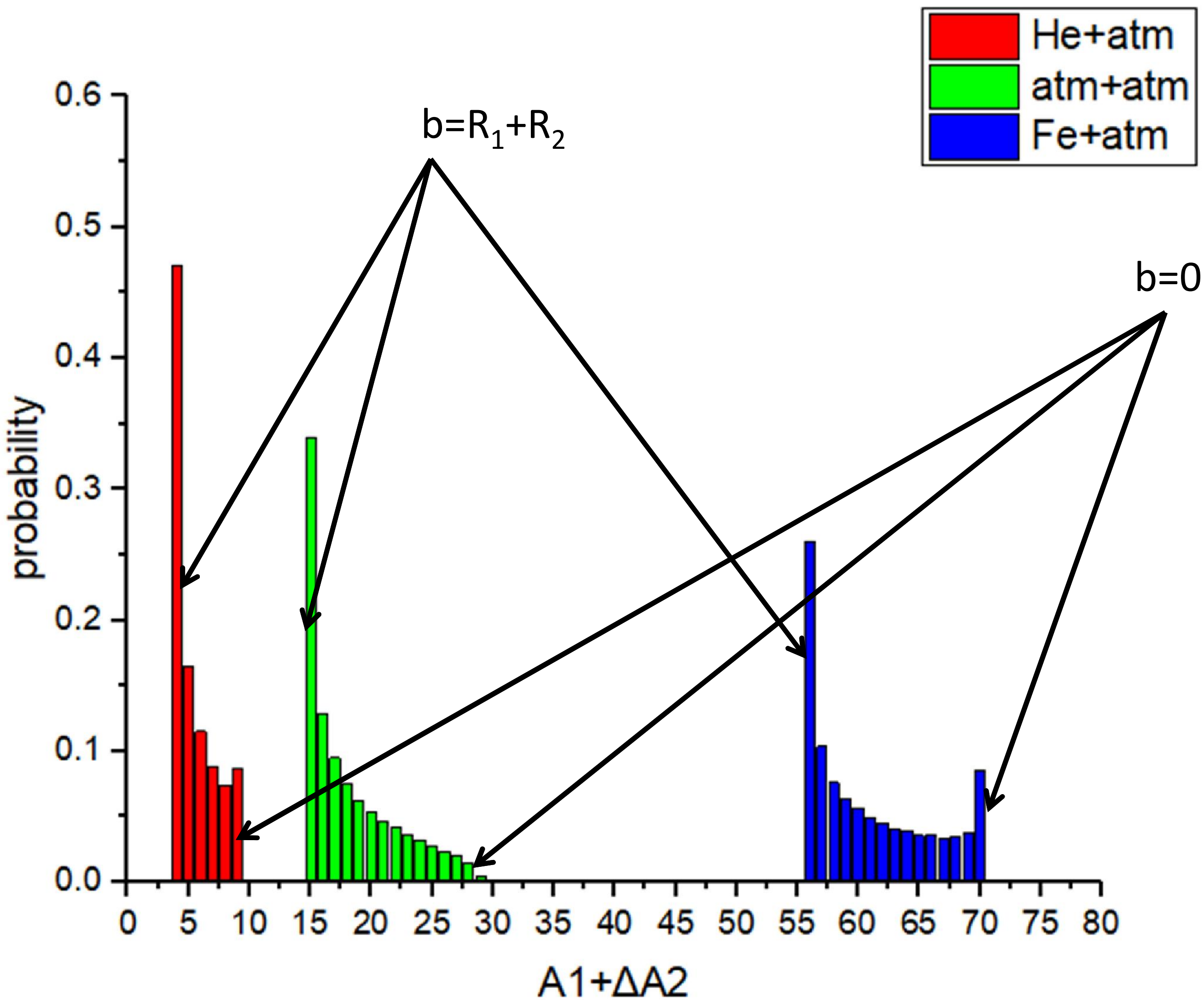


- They determine the energy which we can measure on the Earth's surface
- We consider that the shaded part of the nuclei moves towards the Earth surface
- This number is defined as:

$$A_1 + \Delta A_2$$

- $A_2 = 14.5$

Number of nucleons which fly towards the Earth’s surface after the collision



The speed of the QGP-blob

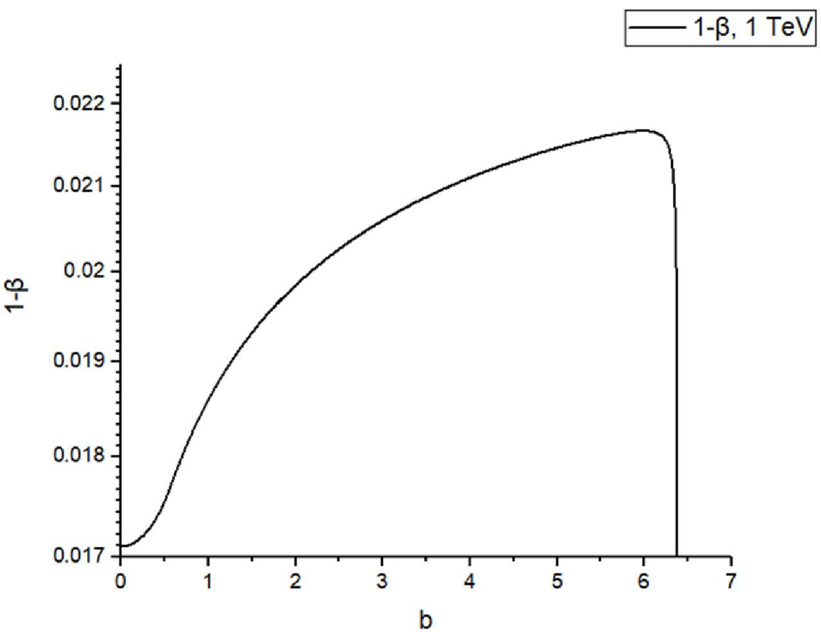
- Let the incident nucleus have energy E_1 and momentum p_1
- The speed of the QGP-blob in the laboratory system is the same as the speed of the Center-of-mass for the interacting number of the nucleons

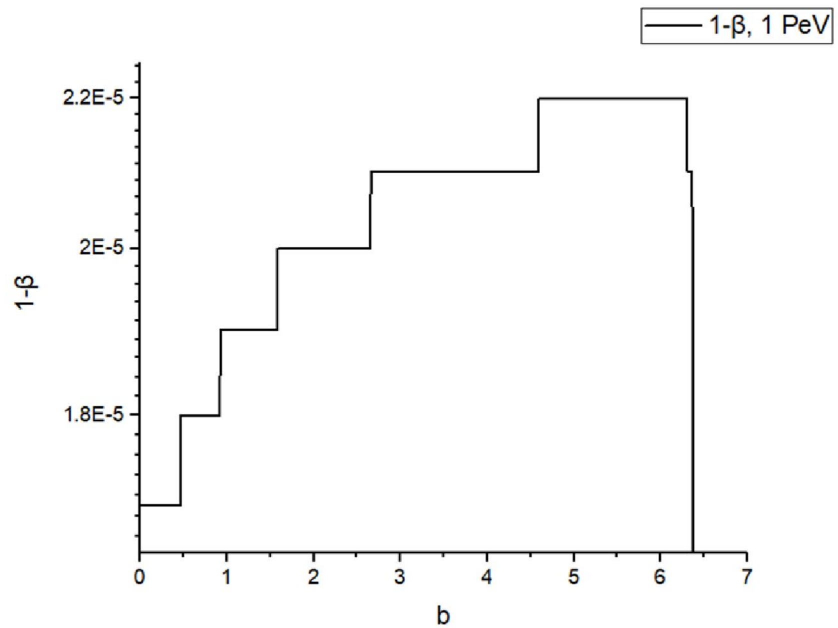
$$\beta_0 = \frac{\Delta p_1}{\Delta E_1 + \Delta m_2} = \frac{p_1 \dfrac{\Delta A_1}{A_1}}{E_1 \dfrac{\Delta A_1}{A_1} + m_N \Delta A_2}$$

The speed of the QGP-blob

- For two equal nuclei the speed remains constant for various b and is the same as the speed of the COM-system of the interacting nuclei:
- $\Delta A_1 = \Delta A_2 = \Delta A_0$
- $\beta_0 = \dfrac{p_1}{E_1 + m_N A_0} = \dfrac{p_1}{E_1 + m_1}$
- Let us consider the collision between an incident nucleus with A_1 = 24 and a target with A_2 = 14.5

The speed of the QGP-blob





Conclusion

- Only 25 % nucleons of each nucleus participate in the collision
- The speed of QGP-blob can be estimated as the speed of the COM-system of the nucleons-participants