Lateral distribution functions of signals in surface scintillation detectors of the Yakutsk Array and new energy estimates of extensive air showers and the energy spectrum of the primary cosmic radiation at energies $\sim 10^{17} - 10^{19}$ eV

Dedenko $^{1,2}$ L.G., Lukyashin $^{3,4}$ A.V., Roganova$^2$ T.M.

1. M.V. Lomonosov Moscow State University, Faculty of Physics
2. Skobeltsyn Institute of Nuclear Physics
3. National Research Center «Kurchatov Institute» - ITEP
4. National Research Nuclear University «Moscow Engineering Physics Institute»

ISCRA 2019
The Yakutsk array
• Oktemtsy village, 50 km south of Yakutsk near Lena river.
  • Area: \(\sim 8 \text{ km}^2\)
  • \(\sim 58\) surface scintillation detectors
  • \(\sim 6\) underground scintillation detectors
  • \(\sim 48\) detectors of the Vavilov-Cherenkov Radiation
    250, 500, 1000 m between detectors
Мотивация
Detectors arrangement in a station

- Cherenkov light detector
- Scintillator & PMT
- Data processing unit
- Receiver & antenna
- Cable connection to central unit

1.5 m
OUTLINE

• 1. Lateral Distribution Function (LDF)
• NEW APPROXIMATION
• 2. New energy estimate
• NEW METHOD OF SIGNAL SIMULATION
• 3. New energy spectrum
• NEW ENERGY ESTIMATES USED
• 4. Fluctuations and energy estimates
• REAL UNCERTAINITIES OF ENERGY ARE LARGE FOR THE ONE DETECTOR
Signals in scintillation detectors
Lateral Distribution Function (LDF)

- Parabolic approximation
- of LDF
- for individual showers have been suggested to take into account fluctuations
Nichimura-Kamata-Greisen function

- Nichimura-Kamata-Greisen function for fitting average lateral distributions of charged particles in the electron-photon cascades:

\[ f(r) = C(s) \left( \frac{r}{r_M} \right)^{s-2} \left( 1 + \left( \frac{r}{r_M} \right) \right)^{s-4.5}. \]

- Here a value \( s < 2 \) has a sense of the “age” parameter and \( r_M \) is the Moulier radius of lateral displacement due to the Coulomb scattering.
J. Linsley’s function

- J. Linsley suggested a function practically of the same type:

$$f(r) = C \left( \frac{r}{r_M} \right)^{-a} (1 + \left( \frac{r}{r_M} \right))^{a-b}$$

- for lateral displacement of signals in scintillation detectors at the Volcano Ranch array.

- In scintillation detectors gamma-quanta produce a main contribution to a signal.

- A. Watson suggested to measure such signals in some experimental units called VEM (Vertical Equivalent Muon).

- So, some conditional (not real) particles were introduced.
Conditional particles

• The energy deposits to total signal in detector (mainly by gammas) are considered as conditional particles which look like muons with energies 10.8 MeV (1 VEM for the Yakutsk detector) due to a suggestion by A. Watson.

• They are not real particles
Fractions of energy deposit to total signal $s$ in detector by $\gamma, e^+, e^-, \mu^\pm$ in the shower with $E=10^9$ GeV

<table>
<thead>
<tr>
<th>$r, \text{m}$</th>
<th>$s(r, \theta), \text{MeV}\cdot\text{m}^{-2}$</th>
<th>$\gamma$</th>
<th>$e^+$</th>
<th>$e^-$</th>
<th>$\mu^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$4,00 \times 10^2$</td>
<td>0.64</td>
<td>0.09</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>600</td>
<td>$3,85 \times 10^1$</td>
<td>0.58</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>1000</td>
<td>$5,63 \times 10^0$</td>
<td>0.47</td>
<td>0.08</td>
<td>0.12</td>
<td>0.33</td>
</tr>
</tbody>
</table>
• At the AGASA array the lateral distribution function:

\[ f(r) = C \left( \frac{r}{r_M} \right)^{-1.2} (1 + \left( \frac{r}{r_M} \right))^{1.2 - \eta} (1 + \left( \frac{r}{r_0} \right)^2)^{-0.6}. \]

• Here \( r_M = 125 \, m, \, r_0 = 1000 \, m \), and parameter \( \eta = 3.97 - 1.79 (\sec \theta - 1) \) was a simple function of the zenith angle \( \theta \).
Yakutsk array function

- At the Yakutsk array the Linsley's function was used:
  \[ f(r) = C \left( \frac{r}{r_M} \right)^{-1} \left( 1 + \left( \frac{r}{r_M} \right) \right)^{1-b}. \]
- Here \( r_M = 79 \, m \)
Parabolic approximation of LDF

- Variables: \( x = \log(r/1\ m), \ y = \log(s(r,\theta)/\ 1\ VEM) \),

- The unit to measure signal is

- 1 VEM=10.8 MeV for **ALL inclined showers**. Signals \( s(r,\theta) \) in detectors for inclined showers were calibrated by GEANT4.

Approximation

- \( y = a + bx + cx^2 \)

- The **LEAST SQUARE METHOD** was used to estimate \( a, b, c \).
Some examples

- FLUCTUATIONS in REAL SHOWERS
- Thin lines – individual showers
- Thick line – Linsley function
Yakutsk data. LDF normalized at 300 m
Yakutsk data. LDF normalized 600 m

\[
\text{Lg}(s(r, \theta)/\text{VEM})
\]

\[
\begin{array}{c}
\text{r, m} \\
5 \times 10^2 \\
1 \times 10^3 \\
1.5 \times 10^3
\end{array}
\]
Yakutsk data. LDF normalized at 1000 m
LDF for shower N27, ○-Yakutsk data, ——parabolic ——Linsley’s type
LDF, ■ - Yakutsk data, — - parabolic approximation
LDF, ■ - Yakutsk data, — - parabolic approximation
Yakutsk data. LDF. Parabolic approximation.

\begin{align*}
\text{Lg}(s(r, \theta) / \text{VEM})
\end{align*}

\begin{tabular}{|c|c|c|}
\hline
\text{Equation} & \text{y = Intercept + B1\text{x}^1 + B2\text{x}^2} \\
\text{Weight} & \text{No Weighting} \\
\text{Residual Sum of Squares} & 0.0979 \\
\text{Adj. R-Square} & 0.92465 \\
\text{Value} & \text{Standard Error} \\
\text{Intercept} & 41.13168 & 19.98077 \\
\text{B1} & -26.89252 & 14.91416 \\
\text{B2} & 4.43695 & 2.77525 \\
\hline
\end{tabular}

\begin{align*}
2.0 & \quad 1.5 \\
1.0 & \quad 0.5 \\
2.4 & \quad 2.6 & \quad 2.8 & \quad 3.0
\end{align*}
Yakutsk data. LDF N27. Parabolic approximation

Equation:
\[ y = 10^{a \cdot (\ln(x) / \ln(10))^2 + b \cdot (\ln(x) / \ln(10)) + c} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>EAS_Signal_Pol2 (User)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>[ y = 10^{a \cdot (\ln(x) / \ln(10))^2 + b \cdot (\ln(x) / \ln(10)) + c} ]</td>
</tr>
<tr>
<td>Reduced Chi-Sqr</td>
<td>2.4646</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.88824</td>
</tr>
<tr>
<td>a</td>
<td>-0.04784</td>
</tr>
<tr>
<td>b</td>
<td>-2.65837</td>
</tr>
<tr>
<td>c</td>
<td>9.44449</td>
</tr>
</tbody>
</table>
Fluctuations of signals

• Estimates of the Poisson fluctuations

• Real simulations without thinning
Poisson deviation $\sigma_p$ of signals $s$ in surface detectors

$\sigma_p, \text{VEM/m}^2$

$E$, EeV

300 m

600 m

1000 m
Relative fluctuations $\sigma_{P/s}$

<table>
<thead>
<tr>
<th>E, EeV</th>
<th>300 m</th>
<th>600 m</th>
<th>1000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.242</td>
<td>0.849</td>
<td>2.32</td>
</tr>
<tr>
<td>0.3</td>
<td>0.141</td>
<td>0.503</td>
<td>1.48</td>
</tr>
<tr>
<td>0.5</td>
<td>0.124</td>
<td>0.402</td>
<td>1.11</td>
</tr>
</tbody>
</table>
NEW energy estimates

1. For each inclined shower
   • with the zenith angle $\theta$ detectors placed in the array plane have different characteristics.
2. The signals in such detectors should be simulated with the help of the GEANT4 and the CORSIKA packages.
   CORSIKA – secondary particles from a shower
   GEANT4 – signals in detectors from these secondary particles
New energy estimates of inclined showers

- \( E_n = a(\theta) \cdot s(600,\theta) \cdot \text{EeV} \)
- \( a(\theta) = a_0 + a_1 \cdot (\sec\theta - 1) + a_2 \cdot (\sec\theta - 1)^2 \)
- The zenith angle \( \theta \).
  - Signals \( s(600,\theta) \) at distance 600 m
  - from the shower core are expressed
    - in units \( E_{VEM} = 10.8 \text{ MeV} \).
- Coefficients \( a(\theta) \) were calculated for showers with \( \theta = 0^\circ, 15^\circ, 30^\circ \) and \( 45^\circ \) and then values of \( a_0, a_1, a_2 \) were estimated.
<table>
<thead>
<tr>
<th>Model</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QGSJETII-04</td>
<td>0.289</td>
<td>0.051</td>
<td>1.34</td>
</tr>
<tr>
<td>EPOS LHC</td>
<td>0.269</td>
<td>0.0347</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Model calibration

QGSJETII-04, EPOS LHC and 6 other models had been calibrated with the help of the atmospheric vertical muon energy spectrum. Simulations and Calibration with PPhDATA.
The primary proton spectrum

$$(\sigma dE_p/dE)_p \cdot E_p^{2.7}$$

$$(\text{GeV}^{1.7} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1})$$

$E$, GeV

28.06.2019  ISCRA 2019

28.06.2019  ISCRA 2019 38
The primary He spectrum
Energy distributions of the primary protons for 3 bins of muon energy spectrum

$\Phi_p(E) \cdot S_{\mu}(E_\mu, E)$

1 - $E_\mu = 106$ GeV
2 - $E_\mu = 1.06$ TeV
3 - $E_\mu = 10.6$ TeV

$E$, GeV
The Yakutsk energy spectrum

• NEW energy estimates had been used to simulate

• the new Yakutsk energy spectrum.
Energy spectra. ●-Yakutsk, △-TA, □-PAO

\[ \log(F \cdot E^3 \text{ } \text{(m}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{eV}^2)) \]

QGSJETII-04
Energy spectra. ●-Yakutsk, △-TA, □-PAO

$\log(F E^3 \text{ (m}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{eV}^2))$

$\log E (\text{eV})$

$E_{\text{POS LHC}}$
The energy uncertainties

• Normal distribution of signals $s$ at fixed energy $E$

$$f(s \mid E) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(s - <s(E)>)^2}{2\sigma^2}\right)$$

• Conditional distribution of energy $E$ at fixed signal $s$

$$\varphi(E \mid s) = \frac{F(E)f(s \mid E)}{\int F(E)f(s \mid E)\,dE}$$

• $F(E)$ — the energy spectrum of the primary particles
Conditional distributions of energy $E$ at fixed signal $s$.

$1 - 0.5\sigma$, $2 - \sigma$, $3 - 2\sigma$
CONCLUSION

FOR THE FIRST TIME IT WAS SHOWN THAT THE YAKUTSK ENERGY SPECTRUM IS COMPATIBLE WITH THE WORLD DATA (TA and PAO)
ACKNOWLEDGMENTS

• Authors are much obliged to

• M.I. PRAVDIN and

• S.P. KNURENKO who kindly gave us part of the Yakutsk data to test new method.

• N.N. Kalmykov and E.A. Osipova are also thanked for some assistance.
• Thanks for kind attention